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A new fuzzy transportation algorithm without converting fuzzy numbers

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ABSTRACT

The ranking function is widely used to convert fuzzy numbers to be crisp on solving fuzzy transportation problems. The converting process can indeed make it easier to play the fuzzy transportation method, but from the convenience, it causes failed in interpreting the results of converting fuzzy numbers. This is because the converting process of fuzzy numbers still has subjectivity values, so it cannot be eliminated, moreover, the ordering can cause incompatible input and output fuzzy numbers resulted. Therefore, the new fuzzy transportation method is proposed by fuzzy Analytical Hierarchy Process to order fuzzy parameters on fuzzy transportation problem without converting fuzzy numbers to crisp numbers, then Algorithm 2 until 6 is used to obtain a fuzzy optimal solution. The advantages of the new proposed method can improve the shortcomings of the existing methods, as well as relevant to solve fuzzy transportation problems in real life.

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1. INTRODUCTION

The transportation problem (TB) was introduced by Hitchcock [1] is one of the well-known linear programming problems. In the globalization era, global economic competition and current market scenario are very competitive so that transportation planning of goods becomes increasingly urgent and broader knowledge related to transportation systems is the key to run a company more effectively and efficiently to obtain minimal production costs by providing maximum services. They ensure that in every move the company runs efficiently and distributing raw materials and finished goods very on time. This is one of the linear programming implementations that can provide a comprehensive framework in planning product distribution.

Practically in real life, the transportation problems are uncontrollable. Distribution costs, the maximum quantity of supplies at the source and the minimum quantity of demands to the destination of an item often occur uncertainty and tend to change from time to time. This is caused by technical factors such as engine damage and failure during production as well as non-technical factors such as uncertain weather and changing market conditions. Therefore, Zadeh [2] presented fuzzy numbers to represent imprecise data such that the value uncertainty of distribution costs, the quantities of supplies and demands are defined as a Fuzzy Transportation Problem (FTB). Zimmerman [3] began the discussion with showed out that the fuzzy LP method always results from a fuzzy optimal solution, such that researchers are interested in discussing the development of Zimmerman's method to solve FTB. For Example, Liu and Kao [4] developed the extension principle which was developed by Zadeh [2] to obtain fuzzy objective value of FTB. Dinagar and Palanivel [5] proposed the solving of FTB with a transportation algorithm to determine the IBFS an MODI method to obtain optimal fuzzy solution is trapezoidal fuzzy number. Kaur and Kumar [6] presented two methods i.e. first method brings out balanced FTB into fuzzy linear programming, then solved it to obtain optimal solution by converting fuzzy linear programming into crisp linear programming problems. While, another method converted FTB to be crisp TB and represented it in the form of FTB table, then solved it to optimal solution by dividing it into four crisps FTB table. Gani et al. [7] presented a simple simplex algorithm is called Arsham-Khan algorithm in which to solve FTB with two phases i.e. first phase is Initial Basic Solution (IBS) of feasible or infeasible are obtained. Next, another phase is generated a feasible solution and to change into IBS. Senthilkumar and Vengataasalam [8] discussed solving FTB with two stages i.e. first stage is the converting FTB to be crisp TB by lower and bound of fuzzy numbers. The second stage is used the simplex algorithm to solve crisp TB. The solving of FTB uses classical methods like Vogel's Approximation (VA), North West Corner (NWC) and Least Cost (LC) have also been discussed by many researchers. For example, Gani and Razak (2006) used VA method to solve FTB which is used after considering compatible α-cut level, then α optimal parameter. Samuel and Venkatachalapathy [9] presented modified of VA method to obtain fuzzy optimal solution in which parameters of supply, demand and cost in the form of triangular fuzzy numbers. Basirzadeh [10] used LC method to solve FTB. He also used the magnitude method to convert fuzzy parameters to crisp. Kaur and Kumar [11], [12] developed the classical method is named generalized methods of VA, NWC and LC to solve FTB. They also use a ranking function to order parameters of supply, demand and cost in the form of trapezoidal fuzzy numbers. Shugani et al. [13] proposed a robust method to transform fuzzy numbers to crisp, whereas the classical method of VA was used to obtain optimal solution and total cost. Kadhirvel and Balamurugan [14] used VA method to get Initial Basic Feasible Solution (IBFS) and presented the fuzzy U-V distribution method to obtain optimal solution. Shanmugasundari and Ganesan [15] presented the solving of FTB without converting to be crisp TB in which VA method and Modified Distribution (MODI) method to find fuzzy optimal solution. Chauhan and Joshi [16] proposed robust ranking to rank fuzzy numbers and VA method to find optimal solution. Sudhagar and Ganesan [17] proposed a ranking score method to rank trapezoidal fuzzy numbers and then the classical method of LC as soon as MODI method to find IBFS and fuzzy optimal solution. Ebrahimnejad [18] developed the method proposed by Kaur and Kumar 's method by used the integral value to order parameters of supply, demand and cost in the form of non-normal trapezoidal fuzzy numbers and also used the classical method of LC to obtain IBFS and MODI method to obtain fuzzy optimal solution. Singh and Yadav [19] presented the intuitionistic fuzzy classical methods of IFVA, IFNWC and IFLC and also used accuracy function to order triangular intuitionistic fuzzy numbers. Ebrahimnejad [20] notated the fuzzy optimal solution by Sudhagar and Ganesan's with numerical example is same. He pointed out that the Sudhagar and Ganesan's method has not obtained a fuzzy optimal solution yet, therefore he gave a non-negative fuzzy optimal solution from the numerical example that is solved by the Sudhagar and Ganesan's method which has less fuzzy transportation costs. Mathur, et al. [21] proposed minimum demand-supply method to obtain fuzzy optimal solution and used ranking function to order parameters of distribution cost, supply and demand in the form of trapezoidal fuzzy numbers. Chakraborty, et al. [22] proposed new operation triangular fuzzy number and ranking function. The new operation was operated on the classical method of VA, NWC and LC were used to find IBFS and also MODI to obtain fuzzy optimal solution. Meanwhile, the ranking function used to rank fuzzy optimal value. Hunwisai and Kumam [23] discussed the robust ranking technique to rank fuzzy numbers and ATM (Allocation Table Method) to obtain IBFS and the MODI method to find a fuzzy optimal solution. Srivastava and Bisht [24] presented dichotomized ranking technique based on interval data and also used ranking function to rank triangular fuzzy parameters, then the classical method of LC used to find IBFS and applied MODI method to obtain fuzzy optimal solution. Dinagar and Raj [25] proposed fuzzy parameters in the form of generalized quadratic fuzzy numbers. They used ranking function to rank fuzzy supplies, fuzzy demands and fuzzy objective value, as well as the classical method of LC to find IBFS. Kumar et al. (2019) proposed a Pythagorean fuzzy approach consisting of type 1 to calculate score value of each fuzzy cost, type II use to calculate score value of fuzzy supplies and fuzzy demands. Meanwhile, LC method to find IBFS and MODI method to obtain fuzzy optimal solution. Another method to solve of FTB like Zero point method presented by Pandian and Natarajan [25] to find the fuzzy optimal solution with transportation costs per unit, quantities of supplies and demands are fuzzy trapezoidal numbers. Hamada (2018) offered a modern method to solve Full FTP by reducing distribution costs. He also used ranking function to rank fuzzy parameters. Balasubraman and Subramanian [26] solved FTP using ranking function.

The solving of FTB by many researchers with several methods was proposed and had been discussed of the papers, there were still disadvantages, like Zimmerman [3]; Liu and Kao [4]; Gani and Razak [27]; Dinagar and Palanivel [5]; Pandian and Natarajan [28]; Gani et al. [7]; Samuel and Venkatachalapathy [9]; Senthilkumar and Vengataasalam [8] provided methods of solving FTB without converting so that the method is not easily applied to real life. Then, Basirzadeh [10]; Kaur and Kumar [6], [12]; Shugani et al. [13]; Kadhirvel and Balamurugan [14]; Chauhan and Joshin [16]; Shanmugasundari and Ganesan [15]; Sudhagar and Ganesan [17]; Singh and Yadav [19]; Ebrahimnejad [18]; Mathur, et al. [21]; Chakraborty, et al. [22]; Hunwisai and Kumam [23]; Srivastava and Bisht [24]; Hamada [29]; Balasubraman and Subramanian [26]; Kumar et al. [30] provided the solving of FTB method by converting fuzzy numbers to crisp numbers using the ranking methods. Converting fuzzy numbers can indeed make it easier to run the classical methods of solving FTB, but from the convenience it causes failed in interpreting the results of converting fuzzy numbers. This is because the converting process of fuzzy numbers still has subjectivity values that cannot be completely eliminated, moreover the converting can cause incompatible input and output fuzzy numbers resulted. Not only that, it ranking method is an error or failed when ranking fuzzy numbers having the same compensation of area.

The paper is proposed a new fuzzy transportation method namely the using of fuzzy Analytical Hierarchy Process (AHP) to order fuzzy parameters on FTP without converting fuzzy numbers, but its method can be applied easily in real life. Next, the modified of the classical method is used to find IBFS and MODI method to determine the optimal fuzzy solution. Furthermore, the new operation for substraction of trapezoidal fuzzy numbers is also presented In addition, the results and discussion are presented with a numerical example as an illustration of the new proposed method implementation

2. THE SHORTCOMING OF CONVERTING FUZZY NUMBERS

The section points out the shortcoming of converting fuzzy numbers methods like ranking function.

Definition 1. Given the mapping of ranking function $\mathrm{H}: \mathfrak{f}(\mathbb{R}) \longrightarrow \mathbb{R}$, $\mathfrak{f}$ is a set of fuzzy numbers which is defined as real numbers set. Let two trapezoidal fuzzy numbers $\widetilde{\mathrm{N}} = (\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4: 1)$ and $\widetilde{\mathrm{M}} = (\varrho_1, \varrho_2, \varrho_3, \varrho_4: 1)$. The ordering ranking of ranking function as follows.

1. if $\widetilde{N} \succeq \widetilde{M}$ then $\mathrm{H}(\widetilde{N}) = \frac{(\varepsilon_1+\varepsilon_2+\varepsilon_3+\varepsilon_4)}{4} \succ \mathrm{H}(\widetilde{M}) = \frac{(\varrho_1+\varrho_2+\varrho_3+\varrho_4)}{4}$,
2. if $\widetilde{N} \preceq \widetilde{M}$ then $\mathrm{H}(\widetilde{N}) = \frac{(\varepsilon_1+\varepsilon_2+\varepsilon_3+\varepsilon_4)}{4} \prec \mathrm{H}(\widetilde{M}) = \frac{(\varrho_1+\varrho_2+\varrho_3+\varrho_4)}{4}$,
3. if $\widetilde{N} \approx \widetilde{M}$ then $\mathrm{H}(\widetilde{N}) = \frac{(\varepsilon_1+\varepsilon_2+\varepsilon_3+\varepsilon_4)}{4} \approx \mathrm{H}(\widetilde{M}) = \frac{(\varrho_1+\varrho_2+\varrho_3+\varrho_4)}{4}$.

Furthermore, Ebrahimnejad [18] presented the linear ranking function of non-normal trapezoidal fuzzy numbers $\widetilde{N} = (\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4: 1)$ as follows

$$\mathrm{H}_{\widetilde{N}}^{\gamma}(x) = ((1-\gamma)\varepsilon_1 + \gamma\varepsilon_4) + \frac{1}{2}((1-\gamma)(\varepsilon_2 - \varepsilon_1) + \gamma(\varepsilon_3 - \varepsilon_4) \quad (1)$$

with $\gamma = [0,1]$ is the index of optimism. γ is used decision-makers to reflect the optimism degree where $\gamma = 1$ is an optimistic decision, $\gamma = 0.5$ is a moderate decision and $\gamma = 0$ is a pessimistic decision

The ranking function is presented by Kaur and Kumar [6], [12]; Mathur, et al. [21]; Chakraborty, et al. [22] on Definition 1 has a shortcoming in converting fuzzy numbers. Example 1 can be seen that the ranking function method was presented by ranking function failed to rank trapezoidal fuzzy numbers and also did not reasonable properties for ordering of fuzzy numbers.

Example 1. Let two the normal trapezoidal fuzzy numbers $\widetilde{\mathrm{N}} = (1,4,9,19)$ and $\widetilde{\mathrm{M}} = (2,5,8,18)$. obvious that $\widetilde{\mathrm{N}} \nsim \widetilde{\mathrm{M}}$. Such that, using ranking function of Kaur and Kumar [6], [12]; Mathur, et al. [21]; Chakraborty, et al. [22] can result $\mathrm{H}(\widetilde{\mathrm{N}}) = 8.25$ and $\mathrm{H}(\widetilde{\mathrm{M}}) = 8.25$. So, the ordering ranking of fuzzy numbers $\widetilde{\mathrm{N}}$ and $\widetilde{\mathrm{M}}$ are equal ($\widetilde{\mathrm{N}} \sim \widetilde{\mathrm{M}}$). While ranking function of Ebrahimnejad [18] by Eq. (1) with moderate of the decision maker or $\gamma = 0.5$ can result $\mathrm{H}_{\widetilde{\mathrm{N}}}^{0.5} = 8.25$ and $\mathrm{H}_{\widetilde{\mathrm{M}}}^{0.5} = 8.25$. Thus, the ordering ranking of $\widetilde{\mathrm{N}}$ and $\widetilde{\mathrm{M}}$ are also equal ($\widetilde{\mathrm{N}} \sim \widetilde{\mathrm{M}}$). Therefore, based on the obtained of the ranking function is presented by Kaur and Kumar [6], [12]; Mathur, et al. [21]; Chakraborty, et al. [22] with $\widetilde{\mathrm{N}} \nsim \widetilde{\mathrm{M}}$ are failed the correctly rank given fuzzy numbers.

3. FORMULATION OF FUZZY TRANSPORTATION PROBLEM

Let consider following FTB.

$$\min \tilde{z} = \bigoplus_{\imath=1}^{m} \bigoplus_{\jmath=1}^{n} \widetilde{\varsigma_{\imath\jmath}} \otimes \tilde{x}_{\imath\jmath} \quad (2)$$

subject to

$$\begin{cases} \bigoplus_{\jmath=1}^{n} \widetilde{x_{\imath\jmath}} \preceq \widetilde{\zeta_\imath} & ; \imath = 1,2,\ldots,m \\ \bigoplus_{\imath=1}^{m} \widetilde{x_{\imath\jmath}} \succeq \widetilde{\sigma_\jmath} & ; \jmath = 1,2,\ldots,n \\ \widetilde{x_{\imath\jmath}} \succeq \tilde{0} & ; \forall \imath,\jmath. \end{cases} \quad (3)$$

where $\widetilde{\zeta_\imath}$ is fuzzy supplies quantity of product at $\imath^{th}$ warehouse, $\widetilde{\sigma_\jmath}$ is fuzzy demands quantity of product at $\jmath^{th}$ object, $\widetilde{\varsigma_{\imath\jmath}}$ is unit fuzzy transportation costs from at $\imath^{th}$ warehouse to at $\jmath^{th}$ object and $\tilde{x}_{\imath\jmath}$ is approximate units of product numbers that should be delivered from $\imath^{th}$ warehouse to at $\jmath^{th}$ object.

If $\bigoplus_{\imath=1}^{m} \widetilde{\zeta_\imath} = \bigoplus_{\jmath=1}^{n} \widetilde{\sigma_\jmath}$ then FTB is called balanced FTB. Otherwise, it is called unbalanced.

4. ANALYTICAL HIERARCHY PROCESS

In this section, we use the ordering method without converting fuzzy numbers to be crisp. Analytical Hierarchy Process (AHP) algorithm is one of algorithms that is widely used by decision-makers, to help solve an assessment problem of several factors. This algorithm was developed by Prof. Thomas L. Saaty as a decision-making algorithm for Multi-Criteria Decision Making (MCDM). The multi-criteria problem in AHP is simplified in the form of a hierarchy consisting of three main components. That is the objective or destination of decision making, assessment criteria and alternatives. The steps in the fuzzy AHP algorithm made is as follows

Table 1. The scale of pairwise comparison matrix

Value	Information
1	$\widetilde{\zeta_\imath}$ as important as $\widetilde{\sigma_\jmath}$
3	$\widetilde{\zeta_\imath}$ little more important than $\widetilde{\sigma_\jmath}$
5	$\widetilde{\zeta_\imath}$ clearly more important than $\widetilde{\sigma_\jmath}$
7	$\widetilde{\zeta_\imath}$ very obviously more important than $\widetilde{\sigma_\jmath}$
9	$\widetilde{\zeta_\imath}$ absolutely more important than $\widetilde{\sigma_\jmath}$
2,4,6,8	when in doubt between two adjacent values

Algorithm 1. Analytical Hierarchy Process

Step 1	Determine fuzzy supplies quantity ($\widetilde{\zeta_\imath}$) and fuzzy demands quantity ($\widetilde{\sigma_\jmath}$) on the FTB.
Step 2	Making pairwise comparison or pairwise matrix for each $\widetilde{\zeta_\imath}$ and $\widetilde{\sigma_\jmath}$. For assessment using the 1-9 Saaty pairwise comparison as shown in Table 1. Next, adds the column pairwise matrix.
Step 3	Calculate the criteria weight (priority vector) i.e (1) normalizing the value of each pairwise matrix column by dividing each value in the matrix column with the corresponding column addition results; (2) calculate value of the row average from the results pairwise matrix normalization.
Step 4	Test the consistency of each paired matrix with the formula for each pairwise matrix element multiplied by the priority value of the criteria. The results are added to each row, then the results with each priority value of the criteria θ_1, θ_2, θ_3,..., θ_n. Calculates the maximum value of λ with formula: $\lambda_{max} = \frac{\sum \lambda}{n}$
Step 5	Calculate the value of the Consistency Index (CI), with formula: CI = $\frac{(\theta_{max}-n)}{n-1}$.
Step 6	Calculate the Consistency Ratio (CR), using the formula: CR $=\frac{CI}{RI}$. If CR $<$ 0.1, then the comparison value pair up on the criteria matrix which is given consistently. If CR = 0.1, then the value pairwise comparisons on the criteria matrix given is not consistent. So if inconsistent, then charging the values at the paired matrix on the criteria element must be is repeated.
Step 7	Arrange matrix rows between criteria whose contents are the results of the calculation process step 4, step 5, and step 6

Step 8 The final result is a global priority as a value used by decision-makers

5. NEW FUZZY TRANSPORTATION ALGORITHM

In this section, we proposed the new fuzzy transportation method without converting fuzzy numbers based on the new classical methods of NWC, NLC and NVA. This algorithm is used to find IBFS and obtained fuzzy optimal solution of FTB.

5.1 Method of Finding IBFS

Based on Eq. (2) and Eq. (3), The steps of finding the IBFS as follows:

Algorithm 2. Balanced FTB

Step 1 Making FTB the formulated fuzzy linear programming problem into the tabular form known as FTB table (FTBT).

Step 2 Create a mathematical model on the real-life of FTB as Eq. (2) and (3).

Step 3 Making FTBT from Step 2.

Step 4 Check whether:

(a) if $\bigoplus_{\imath=1}^{m} \tilde{\zeta}_\imath \approx \bigoplus_{\jmath=1}^{n} \tilde{\sigma}_\jmath$ then next to Step 4;

(b) if $\bigoplus_{\imath=1}^{m} \tilde{\zeta}_\imath \neq \bigoplus_{\jmath=1}^{n} \tilde{\sigma}_\jmath$ then there are two conditions as follows:

- $\bigoplus_{\imath=1}^{m} \tilde{\zeta}_\imath \succ \bigoplus_{\jmath=1}^{n} \tilde{\sigma}_\jmath$ then enter a fuzzy factitious column that has all its $\widetilde{\varsigma_{\imath\jmath}} \approx (0,0,0,0)$. Let $\bigoplus_{\imath=1}^{m} \tilde{\zeta}_\imath \ominus \bigoplus_{\jmath=1}^{n} \tilde{\sigma}_\jmath$ as the fuzzy demand at this fictitious object. Next, go to step 4.
- $\bigoplus_{\imath=1}^{m} \tilde{\zeta}_\imath \prec \bigoplus_{\jmath=1}^{n} \tilde{\sigma}_\jmath$ then enter fuzzy factitious rows that have all its $\widetilde{\varsigma_{\imath\jmath}} \approx (0,0,0,0)$. Let $\bigoplus_{\jmath=1}^{n} \tilde{\sigma}_\jmath \ominus \bigoplus_{\imath=1}^{m} \tilde{\zeta}_\imath$ as the fuzzy supply at this fictitious object. Next, go to step 4.

Step 5 Ordering fuzzy numbers of $\widetilde{\varsigma_{\imath\jmath}}$ by using Algorithm 1.

Step 6 Play on the presented the new classical methods of NWC, NLC and NVA in Algorithm 3, 4 and 4, respectively.

Algorithm 2. Finding IBFS by using NWC

Step 1 Chose cell $(\imath, \jmath)$ of the NNWC in the FTBT and determine a minimum of $\tilde{\zeta}_\imath$ or $\tilde{\sigma}_\jmath$. Next, will come three possibilities as follow:

(a) min $(\tilde{\zeta}_\imath, \tilde{\sigma}_\jmath) \approx \tilde{\zeta}_\imath$, so make on $\tilde{x}_{\imath\jmath} \approx \tilde{\zeta}_\imath$ where $m \times n$ FTBT of the NNWC. Then, neglect row of $\imath^{th}$ to find a new FTBT with $(m-1) \times n$ ordering. Change $\tilde{\sigma}_\jmath$ by $\tilde{\sigma}_\jmath \ominus \tilde{\zeta}_\imath$ in the new FTBT. Next step 2

(b) min $(\tilde{\zeta}_\imath, \tilde{\sigma}_\jmath) \approx \tilde{\sigma}_\jmath$, so make on $\tilde{x}_{\imath\jmath} \approx \tilde{\sigma}_\jmath$ where $m \times n$ FTBT of the NNWC. Then, neglect column of $\jmath^{th}$ to find a new FTBT with $(n-1) \times m$ ordering. Change $\tilde{\zeta}_\imath$ by $\tilde{\zeta}_\imath \ominus \tilde{\sigma}_\jmath$ in the new FTBT. Next step 2.

Step 2 Repeat Step 1 of the new FTBT, until is reduced to be an FTBT 1×1 order.

Step 3 Make on all $\tilde{x}_{\imath\jmath}$ in a cell of $(\imath, \jmath)^{th}$ that is given by FTBT

Step 4 Resulted of IBFS $\widetilde{\varsigma_{\imath\jmath}}$ are $\tilde{x}_{\imath\jmath}$ and Eq. (2), respectively.

Algorithm 4. Finding IBFS by using NLC

Step 1 Find the smallest $\widetilde{\varsigma_{\imath\jmath}}$ in FTBT and determine a minimum of $\tilde{\zeta}_\imath$ or $\tilde{\sigma}_\jmath$. Next, will come two possibilities as follow:

(a) min $(\widetilde{\zeta}_\imath, \widetilde{\sigma}_\jmath) \approx \widetilde{\zeta}_\imath$, so make on $\tilde{x}_{\imath\jmath} \approx \widetilde{\zeta}_\imath$ where $m \times n$ FTBT of the NLC. Then, neglect row of $\imath^{th}$ to find a new FTBT with $(m-1) \times n$ ordering. Change $\widetilde{\sigma}_\jmath$ by $\widetilde{\sigma}_\jmath \ominus \widetilde{\zeta}_\imath$ in the new FTBT. Next step 2

(b) min $(\widetilde{\zeta}_\imath, \widetilde{\sigma}_\jmath) \approx \widetilde{\sigma}_\jmath$, so make on $\tilde{x}_{\imath\jmath} \approx \widetilde{\sigma}_\jmath$ where $m \times n$ FTBT of the NLC. Then, neglect column of $\jmath^{th}$ to find a new FTBT with $(n-1) \times m$ ordering. Change $\widetilde{\zeta}_\imath$ by $\widetilde{\zeta}_\imath \ominus \widetilde{\sigma}_\jmath$ in the new FTBT. Next step 2.

Step 2 Repeat Step 1 of the new FTBT, until is reduced to be an FTBT 1×1 order.
Step 3 Make on all $\tilde{x}_{\imath\jmath}$ in a cell of $(\imath,\jmath)^{th}$ that is given by FTBT
Step 4 Resulted of IBFS $\widetilde{\varsigma_{\imath\jmath}}$ are $\tilde{x}_{\imath\jmath}$ and Eq. (2), respectively.

Algorithm 5. Finding IBFS by using VAM

Step 1 Select the first row of $\imath^{th}$ and choose the two least $\widetilde{\varsigma_{\imath\jmath}}$, then calculate the deviation its and write the obtain it is in front of the row on the right that is said the fuzzy of opportunity $\widetilde{\varsigma_{\imath\jmath}}$ for first row. conveniently. repeat to compute fuzzy of opportunity $\widetilde{\varsigma_{\imath\jmath}}$ $\forall \imath, \jmath$ the columns and also write them in the bottom of the FTBT under convenient columns.

Step 2 Select the highest fuzzy of opportunity $\tilde{\varsigma}_{ij}$ and noticed the $\imath^{th}$ or $\jmath^{th}$ in which this convenient. Determine the $\widetilde{\varsigma_{\imath\jmath}}$ smallest in FTBT. Find minimum of $\widetilde{\zeta}_\imath$ and $\widetilde{\sigma}_\jmath$. Next, will come two possibilities as follow:

(a) min $(\widetilde{\zeta}_\imath, \widetilde{\sigma}_\jmath) \approx \widetilde{\zeta}_\imath$, so make on $\tilde{x}_{\imath\jmath} \approx \widetilde{\zeta}_\imath$ where $m \times n$ FTBT of the NVA. Then, neglect row of $\imath^{th}$ to find a new FTBT with $(m-1) \times n$ ordering. Change $\widetilde{\sigma}_\jmath$ by $\widetilde{\sigma}_\jmath \ominus \widetilde{\zeta}_\imath$ in the new FTBT. Next step 3.

(b) min $(\widetilde{\zeta}_\imath, \widetilde{\sigma}_\jmath) \approx \widetilde{\sigma}_\jmath$, so make on $\tilde{x}_{\imath\jmath} \approx \widetilde{\sigma}_\jmath$ where $m \times n$ FTBT of the NVA. Then, neglect column of $\jmath^{th}$ to find a new FTBT with $(n-1) \times m$ ordering. Change $\widetilde{\zeta}_\imath$ by $\widetilde{\zeta}_\imath \ominus \widetilde{\sigma}_\jmath$ in the new FTBT. Next step 3.

Step 3 Repeat Step 1 of the new FTBT, until is reduced to be an FTBT 1×1 order.
Step 4 Make on all $\tilde{x}_{\imath\jmath}$ in a cell of $(\imath,\jmath)^{th}$ that is given by FTBT
Step 5 Resulted of IBFS $\widetilde{\varsigma_{\imath\jmath}}$ are $\tilde{x}_{\imath\jmath}$ and Eq. (2), respectively.

Algorithm 6. Obtaining fuzzy optimal solution by using modified distribution method

Step 1 The IBFS obtained of FTP using NNWC or NLC or NVA.

Step 2 Introduce $\widetilde{\alpha}_\imath$ and $\widetilde{\beta}_\jmath$ as of variable convenient for every $\imath^{th}$ and $\jmath^{th}$, respectively. In front of $\imath^{th}$, write $\widetilde{\alpha}_\imath$ in row and $\widetilde{\beta}_\jmath$ at the under of $\jmath^{th}$ in column. The simplified calculation, with $\jmath^{th} \approx \tilde{0}$ is the maximum number of allocations row;

Step 3 Determine $\widetilde{\alpha}_\imath$ and $\widetilde{\beta}_\jmath$ others values by using $\widetilde{\varsigma_{\imath\jmath}} \approx \widetilde{\alpha}_\imath \oplus \widetilde{\beta}_\jmath$ for the base of the cell, then determine $\widetilde{\eta_{\imath,\jmath}} \approx \widetilde{\varsigma_{\imath\jmath}} \ominus (\widetilde{\alpha}_\imath \oplus \widetilde{\beta}_\jmath)$ of non-base of cells. Next, will come two possibilities as follow;

(a) $\widetilde{\eta_{\imath,\jmath}} \approx \widetilde{\varsigma_{\imath\jmath}} \ominus (\widetilde{\alpha}_\imath \oplus \widetilde{\beta}_\jmath) \succeq \tilde{0}$ $\forall \imath, \jmath$ So, the resulted of IBFS is done i.e fuzzy optimal solution has been satisfied;

(b) Otherwise, $\exists$ $\widetilde{\eta_{\imath,\jmath}} \approx \widetilde{\varsigma_{\imath\jmath}} \ominus (\widetilde{\alpha}_\imath \oplus \widetilde{\beta}_\jmath) \prec \tilde{0}$ So, obtained IBFS do not finish yet i.e fuzzy optimal solution is not optimal. Therefore, the getting of fuzzy optimal solution is chosen a cell of $(\imath,\jmath)^{th}$ that $\widetilde{\eta_{\imath,\jmath}}$ in which rank is the smallest negative of $\widetilde{\eta_{\imath,\jmath}}$, then make a horizontal and vertical path closed that starts from unchosen base of cell of $(\imath,\jmath)^{th}$. The path can only replace to angle on a base of cell i$(\imath,\jmath)^{th}$ and the path is chosen path must pass through base and non-base cell of $(\imath,\jmath)^{th}$;

Step 4 Give sign ($\oplus$) and ($\ominus$) for closed-loop started with ($\oplus$) for chosen non-base cells. After that, determine fuzzy quantity on cells with signs ($\oplus$) and ($\ominus$). Consequently, will be obtained new FTPT.

Step 5 Repeat steps 2, 3 and 4 for new FTPT until $\widetilde{\eta_{\imath,\jmath}} \geq \tilde{0}\ \forall \imath,\jmath$;

Step 6 Determine the value of fuzzy optimal solution or objective function by using Eq. (2).

6. RESULT AND DISCUSSION

The section of result and discussion are given case study of FTB was taken the processed data's Kaur and Kumar [6], [12]] is used to show the application of new fuzzy transportation method without converting fuzzy numbers. The resulting fuzzy optimal solution is also compared with the existing fuzzy transportation methods like Kaur and Kumar [6], [12]; Ebrahimnejad [18]; Mathur, et al. [21]; Chakraborty, et al. [22].

Case Study: The data of numerical example by [and Kumar2011]. The case used three fuzzy parameters i.e. $\tilde{\zeta}_\imath$, like the approximate fuzzy supplies of the product available at three sources, $\tilde{\sigma}_\jmath$ as the approximate fuzzy demand of the product at three destinations and the approximate unit fuzzy transportation cost of the product from every source to every destination is notated $\widetilde{\varsigma_{\imath\jmath}}$ where $\imath,\jmath = 1,2,3$. All the fuzzy parameters are represented by a trapezoidal fuzzy number and can be seen in Table 1. Find and calculate value of fuzzy optimal solution, such that the total fuzzy transportation cost to be minimum.

Table 2. The recapitulation fuzzy transportation cost ($\widetilde{\varsigma_{\imath\jmath}}$) for product quantities per unit between different sources and destination of case study

	$\tilde{\sigma}_1$	$\tilde{\sigma}_2$	$\tilde{\sigma}_3$	$\tilde{\zeta}_\imath$
$\tilde{\zeta}_1$	(1,4,9,19)	(1,2,5,9)	(2,5,8,18)	(1,5,7,9)
$\tilde{\zeta}_2$	(8,9,12,26)	(3,5,8,12)	(9,10,11,25)	(4,7,8,10)
$\tilde{\zeta}_3$	(11,12,20,27)	(1,5,10,15)	(4,5,8,11)	(4,5,8,11)
$\tilde{\sigma}_\jmath$	(3,5,8,12)	(4,8,9,10)	(2,4,6,8)	

Clearly, $\bigoplus_{\imath=1}^{m} \tilde{\zeta}_\imath \approx \bigoplus_{\jmath=1}^{n} \tilde{\sigma}_\jmath$ = (9,17,23,30), such that the FTB of case study is said balanced FTB. The next step to find and calculate total transportation cost or fuzzy objective function is solved by using a new fuzzy transportation algorithm. Firstly, the ordering of $\widetilde{\varsigma_{\imath\jmath}}$ use Algorithm 1 with pairwise matrix column for supplies i.e $\begin{pmatrix} & \tilde{\zeta}_1 & \tilde{\zeta}_2 & \tilde{\zeta}_3 \\ \tilde{\zeta}_1 & 1 & 3 & 5 \\ \tilde{\zeta}_2 & 0.3 & 1 & 2 \\ \tilde{\zeta}_3 & 0.2 & 0.5 & 1 \end{pmatrix}$ and demands i.e $\begin{pmatrix} & \tilde{\sigma}_1 & \tilde{\sigma}_2 & \tilde{\sigma}_3 \\ \tilde{\sigma}_1 & 1 & 2 & 3 \\ \tilde{\sigma}_2 & 0.5 & 1 & 0.2 \\ \tilde{\sigma}_3 & 0.3 & 2 & 1 \end{pmatrix}$ such that the result of ordering fuzzy numbers without converting $\widetilde{\varsigma_{\imath\jmath}}$ can be seen in Table 3.

Table 3. The ordering fuzzy numbers of case study without converting

	$\tilde{\sigma}_1$	order	$\tilde{\sigma}_2$	order	$\tilde{\sigma}_3$	order
$\tilde{\zeta}_1$	(1,4,9,19)	7	(1,2,5,9)	5	(2,5,8,18)	3
$\tilde{\zeta}_2$	(8,9,12,26)	2	(3,5,8,12)	4	(9,10,11,25)	9
$\tilde{\zeta}_3$	(11,12,20,27)	8	(1,5,10,15)	1	(4,5,8,11)	6

Based on Table 3, it can be seen that solving of FTB can be solved without converting to rank fuzzy numbers so that fails in interpreting fuzzy numbers because subjectivity in the ranking process can be avoided. In addition, the use of algorithm 1 as a ordering fuzzy numbers will not produce fuzzy numbers or $\widetilde{\varsigma_{\imath\jmath}}$ having equal order. Meanwhile, the ordering of fuzzy numbers $\widetilde{\varsigma_{\imath\jmath}}$ by converting fuzzy numbers to crisp numbers using the ranking function still contains fuzzy numbers having the same ranking. Table 4 shows the ordering of fuzzy numbers using the ranking function performed by existing methods i.e Kaur and Kumar [6], [12]; Ebrahimnejad [18]; Mathur, et al. [21]; Chakraborty, et al. [22].

Table 4. The ordering of fuzzy numbers use ranking function

Ranking method	Rank of ς_{ij}	Order of equal rank
The existing methods	$\begin{pmatrix} 8.25 & 4.25 & 8.25 \\ 13.75 & 7 & 13.75 \\ 17.5 & 7.75 & 7 \end{pmatrix}$	$\varsigma_{11} = \varsigma_{13}$ $\varsigma_{21} = \varsigma_{23}$ $\varsigma_{22} = \varsigma_{33}$

Furthermore, the determining of fuzzy optimal solution is used algorithm 2, algorithm 3 of NNWC, algorithm 4 of NLC and algorithm 5 of NVA. Finally, the checking optimally of its fuzzy optimal solution is used algorithm 6 of fuzzy MODI method. we also compared the obtained result of $\widetilde{\varsigma_{ij}}$ is shown in Table 5.

Table 5. The comparison of $\widetilde{\varsigma_{ij}}$ on a case study.

Finding of IBFS	Numbers of iteration	$\min\tilde{z}$
The existing method		
FNWC	2	(31,92,209,504)
FLCM	2	(48,99,222,499)
FVAM	2	(48,106,246,565)
New fuzzy transportation method		
NNWC	1	(33,89,204,491)
NLC	2	(47,95,222,522)
NVA	2	(27,73,189,457)

The case study of fuzzy transportation problem has been solved by using the presented new fuzzy transportation method without converting fuzzy numbers and the fuzzy transportation algorithm proposed by Kaur and Kumar [6], [12]; Ebrahimnejad [18]; Mathur, et al. [21]; Chakraborty, et al. [22].

The ordering of fuzzy numbers has been done by the existing algorithms and new fuzzy transportation algorithm. Table 3 can be seen the ordering of fuzzy numbers without converting can be used. Whereas, the ordering of fuzzy numbers use ranking function still has shortcomings i.e there are fuzzy numbers have the same order can be seen in Table 4. If compared to result, the ordering of fuzzy numbers without converting (Algorithm 1) is better than ranking function was proposed by the existing algorithms

The difference of ordering fuzzy numbers effects also resulting in fuzzy optimal solution. Table 6 shows that the fuzzy optimal solutions ($\tilde{x}_{ij}$) is obtained the FLC method by the existing method and NLC by the new proposed method have different values. These results are caused by differences in the use of ordering fuzzy numbers method.

Table 6. The comparison of total transportation cost on case study 2.

Finding of IBFS	Optimal solution ($\tilde{x}_{ij}$)
The Existing method	$\begin{cases} \widetilde{x_{11}} = (0,0,1,2) & ; \\ \widetilde{x_{12}} = (1,4,5,6) & ; \\ \widetilde{x_{13}} = (0,1,1,1) & ; \\ \widetilde{x_{21}} = (1,3,4,6) & ; . \\ \widetilde{x_{22}} = (3,4,4,4) & ; \\ \widetilde{x_{31}} = (2,2,3,4) & ; \\ \widetilde{x_{33}} = (2,3,5,7) & ; \end{cases}$

New proposed	$\begin{cases} \widetilde{x_{12}} = (4,5,6,7) & ; \\ \widetilde{x_{13}} = (-3,-1,0,1) & ; \\ \widetilde{x_{21}} = (3,4,5,7) & ; \\ \widetilde{x_{22}} = (0,2,2,2) & ; \\ \widetilde{x_{23}} = (1,1,1,1) & ; \\ \widetilde{x_{31}} = (0,1,3,5) & ; \\ \widetilde{x_{33}} = (4,4,5,6) & ; \end{cases}$.

Furthermore, the number of iterations in the Fuzzy MODI method for checking optimization on fuzzy optimal solutions ($\tilde{x}_{u}$) is obtained by the new fuzzy transportation algorithms are 1 for NNWC, 2 for NLC and 2 for NVA. Whereas, the existing method obtained 1,2,2 iteration for FNWC, FLCM and FVAM, respectively. The results showed that the iterations obtained by the new fuzzy transportation method as fast as the existing methods i.e. Kaur and Kumar [6], [12]; Ebrahimnejad [18]; Mathur, et al. [21]; Chakraborty, et al. [22]. However, the resulted of fuzzy optimal value is obtained by new fuzzy transportation method different from is obtained by the existing methods.

7. CONCLUSION

The paper of new fuzzy transportation method without converting fuzzy numbers is proposed to find fuzzy optimal solution and to determine value of total fuzzy transportation cost or objective function of FTB with fuzzy parameter are transportation cost, supplies and demands of the product are trapezoidal fuzzy numbers. the advantages of the new proposed method is presented using a case study of fuzzy transportation problems whose results can improve from the shortcomings of existing methods. The new proposed method makes sense and can be used by decision-makers to solve fuzzy transportation problems in real life.

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